ANALYSIS AND INTERPRETATION OF DYNAMIC RECORDS

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SYNOPSIS

This paper attempts to provide a generalized summary of the problems of analysis and interpretation of the data taken in dynamic tests, particularly with regard to the difficulties ordinarily encountered. It does not include detailed descriptions of methods.

There is a discussion of the economics of the problem, so far as interpretation and analysis are affected by economic considerations. General methods of processing data by tabulation and by graphical methods are given. The problem of reading and transcribing recorded data is discussed and means of simplifying the analyses are mentioned. There is also a brief description of the applications of computing devices and of mathematical instruments in these analyses. The effect of some limitations in equipment are mentioned.

Dynamic testing, as discussed in this paper, is simply the performance of tests under dynamic conditions: that is, while the unknown magnitudes are varying. The term has been used in a more restricted sense, but the broader definition is used here.

The dynamic testing procedure necessarily involves the use of measuring equipment capable of following rapidly changing phenomena, and many applications require the use of multiple-channel equipment, so that simultaneous records of events at different locations may be made. For the most detailed studies, many channels must be used, since often it is not possible to be certain that successive tests are made under identical conditions. It may be conjectured that if sufficient measuring elements were available, and if they were properly placed on the structure to be studied, one test record could disclose all the details of the dynamic behavior of that structure. From the practical standpoint, this is not usually possible, but a moderate number of test records, made with different installations of gages, can approximate the solution.

The procedure used in dynamic testing—that is, the use of multiple-channel recording equipment and of successive tests in different locations on the same structure—may be used in static tests of complicated structures. Although the same number of gages must be applied, there may be a great saving of time by using automatized recording facilities, and when for any reason the test time is limited, or it is difficult to maintain test conditions, the greater cost of the equipment is soon returned. There is a further advantage in taking simultaneous records in that there are sometimes changes in load, or in reaction, not expected and not observed otherwise.

The dynamic testing procedure may be profitably used in many applications. It is, of course, essential in measure-

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ments on moving vehicles, projectiles, and in high-velocity or impact testing. Complete structures may be studied by the application of test forces and recording of reactions. Transmission of energy through material usually, in one way or another, involves dynamic phenomena. In more prosaic terms, buildings subjected to live loads, bridges, highways, machine tools in operation, vehicles, and many other common engineering structures are fit subjects for dynamic

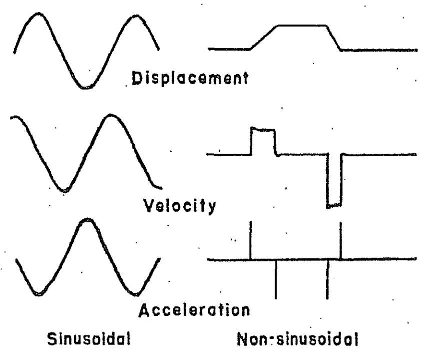


FIG. 1.—Vibration Is a Special Case.

test procedures. Inertia forces can hardly be measured in any other way. Similar techniques may be applied to the measurement of other than mechanical magnitudes—heat, for example.

One of the most common dynamic phenomena, and the one ordinarily chosen for demonstration purposes and for instruction, is mechanical vibration. Vibration is of course a special case of dynamic behavior; the term implies periodicity, and continual return to an initial point. Dynamic behavior in general is not so limited. Vibration study, however, is important in the analysis of dynamic records, if for no other reason than that many primary elements used in dynamic testing are sensitive to one or more of the components of vibration, and their own behavior may influence the record. In Fig. 1 there are two sets of drawings, one showing the response of instruments to sinusoidal vibration, the other the response to a non-recurring, non-sinusoidal displacement. In each case there are instruments responding to displacement, to velocity, and to acceleration. The differences in the records are conspicuous.

This may not be a suitable example, however, since a large part of the dynamic test work in progress at present is done in terms of strain. Still other magnitudes may be measured, and in some work must be measured. The selection of a suitable magnitude and of a suitable sensitive element for its measurement is important, since it may control both the adequacy of the test program and its cost.

ECONOMIC AND OTHER PRACTICAL CONSIDERATIONS

Within the past few years the costs of all work of this type have increased tremendously, and many research projects have become so expensive that they have been abandoned. It is essential, under present conditions, for the directors of any research work to assess the probable costs carefully before making any decisions as to type and scope of work.

Research projects of this kind can be broadly divided into two major classes: laboratory research projects and field test projects. The field test projects can be divided further into exploratory and detailed programs. There is inevitably some overlapping of these classifications.

It is possible with many problems to effect a considerable saving of time and of cost by conducting part (and occasionally all) of a test program in the laboratory. This requires the designing and construction of test pieces which

simulate the actual structures found in the field. Such structures may be small-scale models or they may be portions only of the larger structure; the needs of the problem will dictate which are the better. The principal advantages of the laboratory installation are that there is no cost for transporting and maintaining equipment and personnel at the test location, that tests can be scheduled at any time without the need for waiting until the sought-for phenomena occur, and that the conditions of the test can be more accurately controlled and more readily reproduced. However, corresponding disadvantages are present; the cost of producing test models may be inordinately high and it may prove impossible to scale-down these models accurately, while with some problems actual service conditions may prove the only possible ones giving usable test data.

In general, too, laboratory-type tests can be considered only as exploratory, since usually the only way in which the data can be established as representative is to take some, if not all, of it at the actual location.

Field tests, while usually more expensive are nearly always more satisfactory, and their greater cost is partially offset by the fact that laboratory tests usually must be followed by at least a few tests in the work location to provide checkpoints for analysis. The decision in any instance must be made after considering the several factors mentioned here. The possibilities of the portable single-channel recording device (even the simple mechanical recording extensometer) should not be overlooked. Such devices can be effectively used in exploratory work, and for single-point checking in the field to support laboratory tests.

It is important to note, however, that field tests (whether out-of-doors or merely in some location not in the laboratory) are almost always accompanied by two operating difficulties which occasionally become major items. These are the greater lapse of time involved between taking a test record and seeing its data, and the much greater wear and tear on the test gear. Laboratory-used equipment is normally maintained as it is used; field use does not permit this and involves greater wear as well.

Without regard for the location of the test, there are considerable advantages in certain types of records, and these should be considered before beginning an extensive program. The two principal

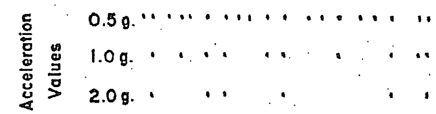


FIG. 2.—Drawing of Accelerometer Record.

forms in which data may be recorded are these: incremental values, in which the data are recorded as occurring only within a few ranges of values; and continuous values, implying that the values are recorded in graphical form as continuous functions, from which individual values can be transcribed.

Incremental-type records have the advantages of nearly always requiring less expensive equipment and of providing data which as recorded have been reduced to a type of statistical average. Such records, however, cannot possibly contain as detailed data, particularly with regard to wave-form and phase changes, as can continuous records taken with the recording oscillograph. In spite of this limitation, incremental-type records are extremely useful.

The most widely used incremental records are those which record in percentage increments of uniform or related size, although some applications are sufficiently well served by equipment which indicates simply that the measured magnitude is within or without a prescribed range. An illustration of such percentage increments is shown in Fig. 2 which represents a section of record from a moving-mass accelerometer. This instrument comprises four spring-suspended masses, each so adjusted that at some specific acceleration value an electrical contact is closed. The contacts may operate counters directly or they may produce their record in some other manner. This drawing is taken from a dynamic record made with a magnetic oscillograph which contained in addition to the usual galvanometer elements a set of small glow-lamps which produced this record. These records must of course be read and transcribed, while the counter readings need only be transferred to the data sheet. However, the more detailed record can disclose coincidences which occasionally serve to identify unknown phenomena. This is also a means of making more recording channels available at little extra cost.

The cruder "pass" and "reject" data may be recorded in precisely the same manner; it may even prove convenient to use an accelerometer indicating only accelerations above some prescribed value. One widely used and well-known example of such an element is the "tilt" indicator on the pin-ball machine.

There is no doubt, however, that more comprehensive and more detailed data can be taken as continuous values on the recording oscillograph; there is, of course, greater difficulty in transcribing the more detailed record. Many applications demand the high resolving power of the continuous record.

Such continuous records may be produced in any one of several forms. Perhaps the most common is the variable-deflection trace, in which the magnitude of the datum is indicated by the deviation of the trace from a reference line. The trace drawn along the top of Fig. 3 shows the form of a variable-deflection record. There are two other well-known forms of oscillograph records in which the value of the unknown magnitude is indicated continuously along the record; they are the variable-density optical record and the variable-width or modulated-envelope record. The variable-width record may be produced by a light-valve type of recorder, or it may be the result of applying a modulated carrier wave to an ordinary recording galvanometer; in either case the general appearance is the same. The center

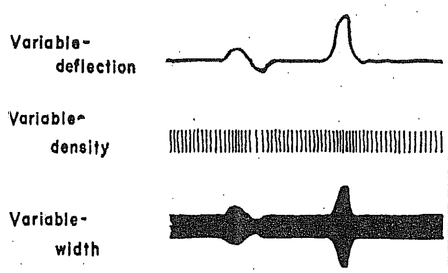


FIG. 3.—Forms of Continuous Records.

trace in Fig. 3 represents a variable-density trace, while the lowest trace is a variable-width or a modulated-envelope trace.

There are other forms of recording devices, producing records containing the same intelligence but of decidedly different appearance. One of these is the pulse-frequency record, in which the frequency of occurrence of impulses is proportional to the magnitude of the measured quantity, and another is the pulse-time modulated record, in which pulses are transmitted continually at a rate which is altered by the intelligence to be conveyed. The accelerometer record in Fig. 2 is a type of pulse-frequency record in which the frequency of the recorded pulses may be considered

an indication of the activity of the system. In Fig. 4 there are drawn examples of a variable-deflection trace, showing a sine wave, a pulse-frequency record of the same sine wave, and a pulse-position modulated trace of the same signal. The pulse-position modulated recording system is especially suitable to radio telemetering of data, since it may be less influenced by atmospheric noise than either of the others. Pulse-frequency records require the least expensive recording facilities, and the variable-deflection oscillograph record ordinarily provides the greatest resolving power of all.

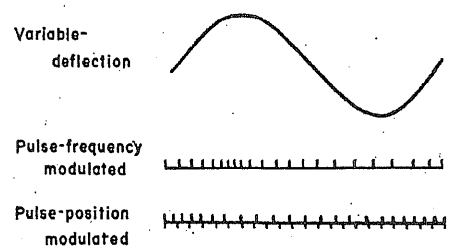


FIG. 4.—Sine Wave Displayed by Three Types of Records.

Exploratory tests may frequently be greatly facilitated by the use of procedures in which more than one phenomenon is recorded on a single trace. Of the many ways by which this can be done, only one example can be given here; it is the use of a single-channel wire or tape recorder to record the outputs of several accelerometers, along with verbal notes on the test. The arrangement consisted of five moving-mass accelerometers, each recording three different acceleration values; each acceleration value for each accelerometer was indicated by a different musical note—15 in all. The record on the wire consisted of a series of short musical notes which could easily be picked off in the laboratory and the data transcribed. Through a microphone, verbal notes on the test could be added when needed. This relatively simple system has the advantages of low cost and bulk, easy operation, long records, and easily obtained equipment and supplies. It cannot, of course, have nearly so great resolving power as more complicated systems. A block diagram of the system is given as Fig. 5.

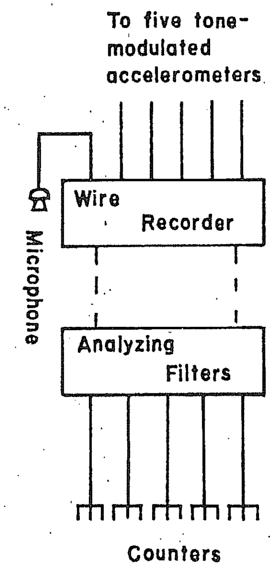


FIG. 5.—Tone-Modulated Accelerometer Recorder.

Since in any dynamic test program, the "reading" of the records is a time-consuming and expensive process, it is desirable to eliminate as much of this task as possible. One means of doing this is through the use of electrical combining circuits. It is possible, for example, to record as a single value the sum or the difference of two of more values, if the primary elements are suitably selected; and if proper recording elements are available, products and quotients are equally easy. Other combinations may be handled with proportionately greater difficulty. Table I is a table classifying the functions of combining circuits, and indicating the locations in which they are most often applied. The selection of desirable combinations and the choice of method must be left to the personnel handling the particular test. The value of such combining circuits in increasing the amount of data that can be recorded with limited equipment should not be overlooked.

It is the function of the research director to evaluate these factors and to decide which procedures are to be followed. He must balance probable value of the data against the cost of the test; he must decide whether field or laboratory tests, or both, should be made; and

TABLE I.—CLASSIFICATION OF COMBINING CIRCUITS.

Process:	Where Performed:
Addition	In measuring elements
Subtraction	In measuring elements
Multiplication	In recorder or computer
Division	In recorder or computer
Trigonometric conversion	In measuring element or computer
Differentiation, integration	In measuring element, in amplifier, or in computer

he should consider the advantages of the several methods which might be used for taking and recording data, their relative costs, and the probability of securing satisfactory data with each. In most instances, the economic factors prohibit the use of the most comprehensive methods; consequently the method which is to be used must be selected with due consideration to the type of problem and to the type of analysis which is required.

ANALYSIS OF GRAPHIC RECORDS

In most instances, measuring and recording the unknown magnitudes is only a small part of the entire problem. Transcription and analysis of recorded data are usually tedious and expensive, and it is not at all uncommon to find that it is economically necessary to take only the cream of the data from a set of records. Careful planning of the entire test program can minimize loss of this kind.

Transcribing and analyzing dynamic records involves three general phases—transcription or "reading" of records, organization of the data for study, and the computation and analysis of data. Each of these phases must be attacked by a different method. Often, a fourth phase must be added—the physical interpretation of the mathematical analysis.

The intelligence contained in graphical records may be classified in these three categories: frequency and magnitude of the unknown quantity; the harmonic content of the unknown, or the interrelations of the various frequencies recorded; and the coincidences of these from one location to another and from one time to another. Within each of these rather general classifications other factors may be considered; for example, in vibration problems the most important items are usually mechanical resonances and damping coefficients. When these are the only data required, one may regard the recorded data from a more restricted viewpoint than otherwise might be permissible. Effectively, this simply means that short-cut methods may be used. Such short-cut methods may involve recording equipment which is specially sensitive to certain frequencies known to be critical; or, more effectively, equipment may be employed in the laboratory to analyze records including all available frequencies. Harmonic analyzers, for example, may be employed to plot the entire range of frequencies and their amplitudes; slightly more complicated analyzing devices may show not only the frequency-*versus*-amplitude relationship but the phase relations between the com-

ponents, and even the phase shifts from one gage location to another. These devices have long been considered too expensive and too complicated for the industrial laboratory, yet some of the simpler forms can be procured at a cost of only a few hundred dollars. Most research laboratories can even extemporize such equipment. Figure 6 is a drawing of

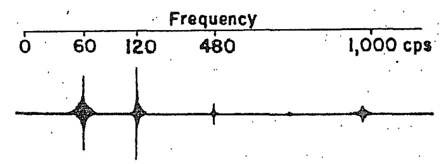


FIG. 6.—Record of Harmonic Analysis.

a record taken with such an extemporized equipment; it shows the harmonic content of a vibration pattern, expressed as a variable-amplitude oscillograph record. The harmonic content usually will disclose the source of the energy.

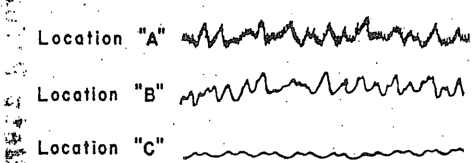


FIG. 7.—Characteristic Vibration Records.

It is not uncommon, too, for the usable data in a specific problem to be presented in a series of records, each involving a "key" frequency, or a tracer frequency. That is, some specific frequency, known to be an indication of difficulty, may be sought for in a number of locations; its relative magnitude, and its phase at these locations, may show the behavior pattern of the structure or disclose the source of the disturbing frequency. As an example of this, Fig. 7 may be considered; in this drawing there are three oscillograph traces, each a record of vibration at a different location. In each there is a strong component at one frequency, and a strong second harmonic superimposed on it. There are other disturbing frequencies in two of the traces, but through other tests they were shown to be subsidiary. The harmonic analysis showed the frequency components; the actual frequency and the phase relationships showed that in this case the magnetic forces within the field windings of a motor were producing mechanical vibrations in the associated equipment—and they also showed a way of eliminating the trouble.

More often than not, it is the coincidences between phase, magnitude, and frequency at different locations and at different times which indicates the answer to a problem involving dynamic conditions. Such coincidences may appear in any one of a large number of forms; they can occasionally be disclosed by tabulating maximum values from an oscillograph record, or they may be plain to the eye when a multi-trace record is examined, as in the correlation of seismograph records. There are many types of such coincidences; this paper can only mention their existence.

A major difficulty in many research programs is due to the extremely large amount of data which can be taken in a short time with multi-channel recording equipment. These data are in the form of oscillographic or similar records, and while they may be preserved indefinitely in this form, they must be transcribed before analysis. The mere task of transcribing such data to work sheets usually takes more man-hours than the experimental work. There are mechanical and electrical aids to the reading of such records, but they have seen little application as yet. The incremental data taken with counter-type equipment is not so handicapped, and such equipment is deservedly popular for this reason. Unfortunately its application is limited.

Undoubtedly at present the most widely used method of reading records is the visual one. This is simply the examination of a record by eye, taking measurements from it by means of some sort of scale and recording the data as read on work sheets. This work may be made easier by using scales graduated directly in units of the unknown magnitude; and there are commercially-made magnifying and projecting devices which make the record easier to see, but at best it is a tedious business, and the human error may become large. The method does have the advantage, however, of being applicable either in the field or in the laboratory, and it is virtually indispensable for exploratory field tests when the rough data being taken must be used in the guidance of succeeding tests.

Visual transcription of data may be detailed and comprehensive, if an experienced worker does it, or it may be cursory and incomplete. Intensive reading of such records requires experience, and is tedious; but a skilled worker can read off not only magnitudes but much data with regard to frequencies, phase relations, and interferences between phenomena.

Many records, especially in vibration studies, may be badly contaminated by frequencies which though always present have no significance in the analysis. In some installations it may not be practical to eliminate the effect of all 60-cycle signal. A skilled record-reader can transcribe the desired intelligence from a badly contaminated record—one which to a less highly-skilled man would appear hopeless. It is always better to have clean, straightforward records, but the inability to produce them does not always prohibit completing a test.

Drawings of three common types of contaminated records are shown in Fig. 8. The first, at A, is a record of a low-frequency sine wave contaminated by a higher frequency. This is easily resolved by drawing the envelope, drawing the average line through its center, and reading this line. The second, drawn at B, is a record of a transient, recorded on a trace heavily contaminated with a lower frequency. Perhaps the most straightforward method of handling this is to plot the contamination, as at C,

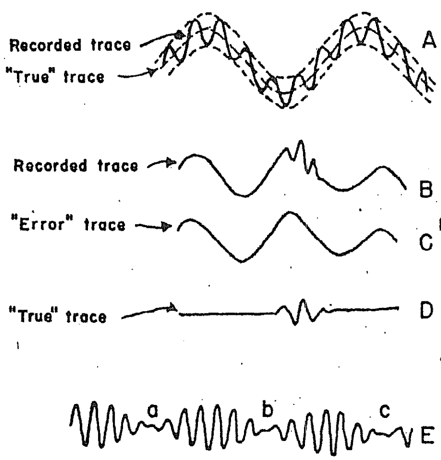


FIG. 8.—Analysis of Contaminated Wave-Forms.

subtract it from the record, and plot up the true value, as at D.

More confusing patterns may appear when two or more frequencies differing by only a small amount are recorded. The usual consequence is a record of interference frequencies, or beat frequencies. One such record is drawn at E. There is an ambiguity in this record which may not be noted at first glance; the wave length of the beat frequency is the distance from a to c, rather than that from a to b.

These are of course only simple examples; the same principles are used in interpreting the more complicated traces

that are recorded when the combined signals contain harmonics.

There are a number of instrumental methods for the transcription of such test records, but none of them has been widely used. Primarily, this is because the devices have nearly always been built to analyze variable-density optical records, or have been highly specialized systems built for some very special application—usually one in which cost was no problem to the designers. So far, there have been no general-purpose instruments of this kind.

Such instruments could be built if the demand existed. Working models have been made and tested, which can read off the maxima, the rates-of-rise, the totalized duration, and even other quantities from any reasonably good oscillographic record. The exploratory system previously mentioned, and illustrated in Fig. 5, is especially susceptible to such "instrumental reading." Most harmonic analyzers are capable of taking a record and "reading" it into their mechanism, if it is presented to them in the proper form.

There has been, perhaps, more development work done on field instruments which produce their records already converted to statistical averages and displayed on counters, than on instruments which can transcribe data from the more detailed continuous record. In applications such as ride-roughness recorders, fatigue testing, and shock-resistance for packages, devices have been developed which can measure and indicate the magnitudes in this manner. The applications, of course, are those in which it has been found practical to correlate such data to actual conditions, and to determine "indices of damage."

Not until the numerical values of the data have been transcribed to work sheets can one begin the detailed process of computation and analysis. Even then, for effective study, the numerical data must be suitably arranged.

The nature of the problem, and the kind of magnitudes being measured, will determine the manner in which the data should be tabulated for study. In vibration problems—frequently the simpler ones—one usually begins by tabulating frequencies against amplitudes, and harmonic content in detail against potential sources of energy. In ride-roughness study, the incidence of too-high rates-of-change of acceleration will be tabulated against vehicle speed for each condition. As an indication of lading-damage, the incidence of accelerations of certain values will be plotted against train speed—and so on. The selection of proper tabular headings is an art, to be learned primarily from experience.

Before actual analysis has proceeded far, it is usually desirable to assess the quality of the recorded data. Each series of records should be classified as to the probable error from operating conditions, from instrument idiosyncrasies, and so forth. Poor records (in the photographic sense, or from the standpoint of being difficult to read) should be so marked. An experienced worker can often detect instrument trouble from examining a record, without seeing the instrument.

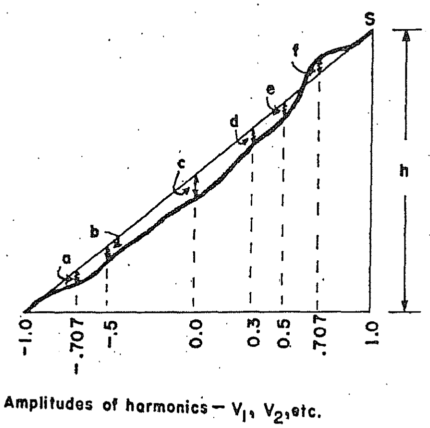
As for the actual methods to be used in the computation—there are a great many, and most of them are adequately described in the textbooks dealing with the treatment of experimenal data. In some respects, however, there are fundamental differences between static and dynamic work, and this deserves some discussion.

Dynamic measurements, from their very nature, are ordinarily subject to greater errors than are anticipated in static measurements. First, because it is not possible to repeat readings, to average them, or to discard them if there is obvious error. Second, because the equipment is intrinsically less accurate because it must respond to rapidly-changing values. And third, because in many types of work the rate-of-change is as important as the magnitude; thus there are two magnitudes being measured by a single instrument instead of one. Part of this loss of accuracy is regained through the introduction of greater amounts of data into the analysis and through the use of statistical methods. On the other hand, too energetic application of statistical methods has been known to conceal intelligence lurking behind small variations in raw data.

Graphical procedures are used in nearly all analyses, either as a method sufficient in itself or as a preliminary to other methods. The qualitative graph, in which a magnitude is plotted against time or against some other magnitude, is an effective and a convenient means for the initial examination of experimental data. It is also a means of exploring for correlation, or stochastical relation, rather than functional relation. The quantitative graph is more often prepared as a convenience in further computation, since its function is to display a mathematical relationship. Their difference is one of purpose rather than of appearance; the qualitative graph is intended to present a qualitative picture while the quantitative graph is intended to serve as a quantitative tool.

Given a quantitative graph, it may be necessary to employ some procedure to fit a curve to it. One usually begins this process by trying different coordinate axes. Polar plots and special statistical forms are often used. If none of these serve, then it will be necessary to apply the procedure of summing-up a set of simple relations, in principle a method of successive approximations. This procedure is described in many texts, and no more need be said here than that it is a means of determining mathematical relations from experimental data.

In analyses of experimental data, it is important to remember that the relation between two magnitudes may not be complete, or perfect. A partial dependency is called a correlation, and the



$$V_1 = \frac{h}{2} + V_3 - V_5 + V_7 \qquad V_2 = \frac{e-b}{3} + \frac{c-a-f}{4}$$

$$V_3 = \frac{e-b}{3} \qquad V_4 = \frac{c-a-f}{4}$$

$$V_5 = \frac{e-b}{3} + \frac{a-f}{2.828} \qquad V_6 = \frac{c}{2} - V_2$$

$$V_7 = \frac{f - 1.82V_2 - 1.092V_3 + 0.655V_4 - 0.699V_5 - 0.751V_6}{1.146}$$

FIG. 9.—Simplified Harmonic Analysis.

term describing the degree of dependency is called the correlation coefficient. Dynamic test data very frequently display correlation between three or more magnitudes; the methods used in the analysis must be capable of discovering such relations. There are several available methods.

As a first step, a qualitative graph may show that two magnitudes seem related; the plotted points define, roughly, a smooth curve. The width of the band of

plotted points is inversely proportional to the correlation between the magnitudes; if the dependency is perfect the points will define a smooth and definite curve—only the errors in the data will serve to broaden the band of uncertainty. There are methods by which the correlation coefficient between any two quantities may be established.

A graphical method by which harmonic analyses may be easily performed is shown in Fig. 9. This system is especially applicable to exploratory tests, since the data may be taken and the analysis performed with no more equipment than a measuring element, a cathode-ray oscilloscope, and an oscillator providing a sine wave of frequency equal to that of the unknown. The procedure consists of applying the unknown wave-form to the vertical plates of the oscilloscope, and a sine wave of the same frequency and phase to the horizontal plates. The result is a single-line pattern somewhat like that drawn in Fig. 9, and this pattern may be transcribed by any means desired—even to tracing it on thin paper with a pencil.

The procedure for computation requires only simple arithmetic. The coordinate axes are drawn in, the abscissa divided as shown, and the maximum ordinate measured. The slant line from -1.0 to S is drawn, and the distances a, b, c, d, e, and f are scaled off, calling distance down from the slant line negative, and up positive. The values thus determined are substituted into the equations shown, and the amplitudes of the harmonics, V_1, V_2, V_3, etc., obtained.

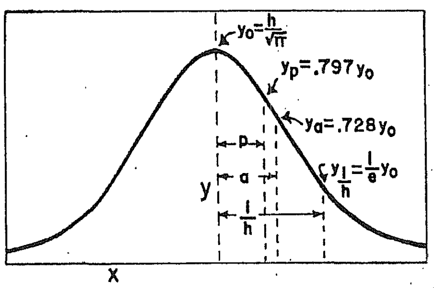
Graphical methods are often not sufficiently precise for analysis of even moderately complicated structures, so that it becomes necessary to employ tabular means of computation. Many of these are almost purely statistical. Procedures for refining experimental data by least-squares methods, by weighting of values, and the like are well known; in these analyses, however, the quality of the records, as determined previously, should be considered. Statistical methods are most often applied to test the suitability or accuracy of relations derived either graphically or through study of correlations. In this process the determination of the statistical "probable error" is usually necessary.

The term "probable error" is perhaps unfortunate. The value referred to is really the "probable deviation from the observed mean," and it is of considerable interest since it can indicate the quality of the work, the reproducibility of test conditions, the uniformity of operation of the entire system, and perhaps the probable errors in the measurements.

The "probable error" is based on the frequency distribution curve, which is prepared by plotting the number of individual values against their magnitudes. The width of the resulting pattern indicates the deviation from the mean, and the "probable error" can be computed, assuming normal frequency distribution. A continuation of the same mathematical process provides two other values, of perhaps greater utility; they are the criterion for rejection of data and the index of precision.

There are several differently derived criteria for the rejection of data, but all have the same purpose; they define the maximum deviation which may be considered permissible. All data with larger deviations from the mean should be discarded. An index of precision is simply a term describing the probability by which a single datum will differ from the mean by a certain amount. Both of these are primarily of statistical interest and are of greater value when large amounts of data, showing low correlation, must be worked up. Figure 10 is a normal frequency distribution curve, showing the average deviation, the "probable error," and the precision index of Gauss.

Another mathematical method, which can be used even when there is not a sufficient bulk of data for the purely statistical methods, is the Fourier analysis. This provides a mathematical solution for a recorded curve containing unknown frequency components. The Fourier analysis is based on the fact that any periodic function, however complicated, can be resolved into a summation of a number of individual sine waves. There are several procedures, all requiring the



y = No. of observations
x = magnitudes of observed values
p = "probable error"
a = average deviation
$\frac{1}{h}$ = precision index

FIG. 10.—Normal Frequency Distribution Curve.

measurement of a number of values on a relatively short length of the record—that is, a number of individual measurements, each of a different point on one cycle of the unknown wave form. Statistical procedure requires a relatively large number of individual values for the same datum. Consequently, the Fourier analysis can be used when only a short record is available—only a few cycles of a repetitive wave form, or, if desired, a single cycle of a non-recurring wave form can be analyzed.

Before ending the description of mathematical methods, it may be in order to mention the tabular methods for taking derivatives and definite integrals, which are not nearly so well known as the graphical ones, but occasionally more applicable.

Of the mathematical instruments used in routine work, only the polar planimeter now has a really wide acceptance. This instrument gives the definite integral of a function; the integrator, which can give in addition to the area the first and second moments, and the integraph, which produces the integral curve of a function, are seldom found except in specialized laboratories. Electrical integration and differentiation in the recording equipment have become almost standard practice; the result, however, is not quite the same.

Other mathematical and mechanico-electrical instruments which might be mentioned are the mechanical harmonic analyzer, the electrical or heterodyne harmonic analyzer, and the optical and photographic harmonic analyzers. Instruments of all three types have been known for many years but never have received very extensive use. They deserve more attention from the industrial laboratories. These instruments occur in many forms, fitted to a number of different needs; there is not space here for any detailed descriptions.

In addition to the instruments mentioned above, there are now available a number of electrical and electronic devices for the solution of mathematical problems, particularly those involving a set of simultaneous equations, and any problem which may be solved by a method of successive approximations. These range from the simple to the extremely complex and expensive.

There are also the well-known calculators which are capable of the customary arithmetic operations and find wide use in all branches of research. They need no description.

INTERPRETATION OF DATA

In general, the two questions to be answered by a research project involving dynamic measurements are "What is happening?" and "What harm is being done?" If the problem is to develop a new product, the first question is important; if there is trouble with a present product, it is the second. Frequently both questions must be answered.

There is usually a statistical answer to the question "What harm is being done?" An exploratory series of tests may establish that strains of a certain magnitude will cause a certain amount of damage; then a routine series of tests will be scheduled to determine the frequency of occurrence of such strain values under each of several sets of conditions. These frequencies are then expressed as "indices of damage," or perhaps the structure or material is described as possessing such-and-such a "resistance factor." More often than not, there is no specific index of damage; there are two or more correlated factors which together describe the condition. It is in the solution of problems whose answers can be expressed in this way that the counter-type instruments are best applied.

As to "What is happening?", there is not usually so simple a means of expressing the final solution. It may be clear from casual observation that there is vigorous activity in some part of a structure; the source of energy causing that activity may be in some other part, or even outside the structure. It may be tracked to its source by measuring and comparing frequency and magnitude at a number of locations, and determining the resonant frequency and the damping of several parts of the structure. Or the procedure may be quite different, in detail; the underlying principle will be much the same. The common factors in mechanical dynamic phenomena are frequencies, amplitudes, and phase, whether they appear as strain, displacement, velocity, or acceleration. In electrical phenomena there are analogous quantities, and in thermal phenomena. Or, for that matter, in physiological behavior.

SUMMARY

Because of the large scope of the subject, and the limited time, this paper could not hope to give more than a generalized summary of the subject. It is hoped that it will serve as a means of guiding the interested person to more detailed information, which is available in the literature. No apology is offered for the complete lack of specific technical data; the discussion is intended for the man whose interests are general, and for the director of research, who needs to know what can be done, and depends upon detail workers to do it.

DISCUSSION

Mr. W. N. Findley[1] (*by letter*).—I was pleased to see such a complete outline of this important and difficult subject. It was a disappointment, however, to note the complete absence of technical detail. It is to be hoped that an additional paper may be expected of the author in which the most practical methods will be treated in detail. In the meantime it would add greatly to the usefulness of the paper if it were copiously sprinkled with references to sources of detailed information.

Mr. T. J. Dolan[2] (*by letter*).—Mr. Roberts has presented the general aspects of the problems attendant to transcribing the results, eliminating contamination, and assessing the quality of recorded data from various types of measuring systems. I should like to heartily endorse his statement that, "Dynamic measurements from their very nature are ordinarily subjected to greater errors than are anticipated in static measurements." Because of this intrinsic difficulty, a careful analysis of the results requires some knowledge of the accuracy and precision of the associated equipment used in obtaining the record and of the calibration methods employed to appraise the value of the magnitudes involved. With particular reference to deflection measurements or to amplitude-modulated systems, many errors may be introduced by instability of the electronic circuits and their components, by cross-talk between channels of measurement, by interaction or electrical losses between various types of gages, and by beats with spurious sources of alternating current or by general electrical "noise."

These difficulties with electrical circuits immediately raise a question as to the reliability of the record and of the probably accuracy of the calibration method. If calibration is accomplished by recording the static response of the equipment (such as by employing a static electrical potential, or a resistance change, etc.) one has no assurance that the dynamic response is of the same nature as (or proportional to) the static calibration. Furthermore, the relative phase shift between two signals for which a comparison is desired (as well as the relative frequency response) may be altered by differences in the associated electrical circuits involved in the measurement, and hence may not be accurately represented in the final record. It is only through careful advance planning and repeated checks on the performance of the equipment that one can analyze the final result with an assurance that it accurately represents the quantities desired in their correct wave form and time sequence. For many types of measurement of mechanical phenomena of short time duration there are no commercially available instruments that are capable of accurately recording

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the desired data. Since Mr. Roberts has had a great deal of experience in this field, it might be helpful if in his closure he could point out a few of the pitfalls that should be avoided in the interpretation of records in view of these many possibilities of difficulties arising from the inadequacies of instrumentation for dynamic measurements.

After careful planning and development of equipment, proper placement of gages or pickup units, precise calibrations and careful analysis, one may usually arrive at a satisfactory answer to the question, "What is happening?" But as the author points out, there is often no specific index to measure "damage"; hence it becomes extremely difficult to answer the question, "What harm is being done?" Thus there still remains the broader question as to just what the results mean as a criterion of the "technical worth" of the component tested in terms of its service requirements. This technical worth might be made up of: (1) the suitability of the component to perform the service expected of it in a satisfactory manner, and (2) the durability and stability of the article when used for that service condition.

The final interpretation of the significance of what is happening must rely upon knowledge of the behavior of materials and past experience with similar components in service, which is a field entirely separate from the subject of the paper. However, in planning the original measurements, one must have sufficient knowledge of the service requirements and of these "technical worth" factors to insure obtaining recorded data that may be directly appraised in terms of the knowledge available regarding the fundamental behavior which measures the nearness of impending damage. In short, the measurements must serve as adequate indices of the fundamental structural or mechanical actions.

Mr. Howard C. Roberts (*author's closure*).—The comments by Messrs. Dolan and Findley are much to the point; as a matter of fact others have raised the same questions. Unfortunately there are no simple and direct answers to these questions.

Mr. Dolan points out that the application of a known resistance change (or a known potential) to the input of a measuring channel as a calibrating procedure does not necessarily produce the same indication on the record as would a similar magnitude through the gage circuit itself. This statement is contrary to much established practice; however, it is quite true. In the general case one cannot be certain that a resistance change applied to a strain-gage bridge will cause the same magnitude and kind of unbalance as would a corresponding resistance change in a gage due to strain. In most specific cases it can be shown that the difference is small enough to be ignored, but in some exceptional cases appreciable errors occur.

Phase errors and errors due to inadequate frequency response of the equipment may usually be anticipated. Good practice requires that the problem first be analyzed so that the probable frequencies to be encountered may be estimated; then the recording equipment must be made responsive to all these frequencies. One can be fairly certain that all frequencies are recorded if none of those appearing on the record approach the limits of response, but usually not otherwise. It is desirable to make frequent checks of the frequency and phase response of the equipment, since both amplifiers and recording galvanometers can experience changes during service.

There is little material in the literature which is useful in showing the performance of oscillographic recording systems. Instruments for measuring steady-state values, and for quite slowly varying magnitudes (recording potentiometers, ink-writing milliammeters, and the like) have been thoroughly discussed in trade and other publications. The more complicated systems involving sensitive measuring elements in conjunction with electronic amplifiers and recording oscillographs have not had such detailed treatment. The average purchaser is likely to take the manufacturer's specifications and be content. This is usually satisfactory if the entire system is supplied by one manufacturer, but if the several elements of the system come from different sources, some rather surprising troubles may appear. In any case, the equipment must be properly maintained and adjusted. For this, a technician trained in this type of work is needed.